AL08

Numerical Algebraic Geometry

Geometría Algebraica Numérica

Zenbakizko Geometria Aljebraikoa

Organizers **Organizadores** **Antolatzaileak**

Margaret H. Regan
(College of the Holy Cross)

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(Johns Hopkins University)

Description **Descripción** **Deskribapena**

Numerical methods are among the fastest computational tools for solving problems in science. Algebraic geometry is no exception, and, as of today, the fastest methods for solving computational problems in this area are numerical. However, the need for reliable numerical methods and the current computational challenges demand the development of new analyses and numerical techniques that can fully exploit the structures we find in the problems from computational algebraic geometry. In this session, several of the latest advances in numerical algebraic geometry will be presented.

Los métodos numéricos se encuentran entre las herramientas computacionales más rápidas para resolver problemas en ciencia. La geometría algebraica no es una excepción y, a día de hoy, los métodos más rápidos para resolver problemas computacionales en esta área son los numéricos. Sin embargo, la necesidad de métodos numéricos fiables y los retos computacionales actuales exigen el desarrollo de nuevos análisis y técnicas numéricas que puedan explotar plenamente las estructuras que encontramos en los problemas de geometría algebraica computacional. En esta sesión se presentarán varios de los últimos avances en geometría algebraica numérica.

Zenbakizko metodoak zientziako problemak ebazteko tresna konputazional azkarrenen artean daude. Geometria aljebraikoa ez da salbuespen bat, eta, gaur egun, eremu honetan problema konputazionalak ebazteko metodorik azkarrenak zenbakizkoak dira. Hala ere, zenbakizko metodo fidagarrien beharrak eta egungo erronka konputazionalek analisi eta zenbakizko teknika berriak garatzea eskatzen dute, geometria aljebraiko konputazionalaren problemaetan aurkitzen ditugun egiturak ustiatu ahal izateko. Saio honetan zenbakizko geometria aljebraikoan egindako azken aurrerapenetako batzuk aurkeztuko dira.

MSC Codes **Códigos MSC** **MSC Kodeak**

65H14
(primary)

14Q65
(secondary)

Slots **Bloques** **Blokeak**

1.B (Aula 0.5); 1.C (Aula 0.5)

QR Code **Código QR** **QR Kodea**

Session Schedule **Horario de la Sesión** **Saioaren Ordutegia**

M14 | 15:00-15:20 | 0.5
SAGBI detection and SAGBI homotopy
Viktoriia Borovik (Osnabrück University)

M14 | 15:30-15:50 | 0.5
Singularities of spectrahedra
Khazhgali Kozhasov (Université Côte d'Azur)

M14 | 16:00-16:20 | 0.5
Generalized real monodromy
Margaret H Regan (College of the Holy Cross)

M14 | 16:30-16:50 | 0.5
Effective homology and numerical integration of periods
Eric Pichon-Pharabod (MPI MiS Leipzig)

M14 | 17:30-17:50 | 0.5
Mukai lifting of self-dual points in $\mathbb{P}^6$
Leonie Kayser (MPI MiS Leipzig)

M14 | 18:00-18:20 | 0.5
Preconditioning via Geodesically Convex Optimization
M. Levent Doğan (Ruhr Universität at Bochum)

M14 | 18:30-18:50 | 0.5
Typical ranks of random order-three tensors
Sarah Eggleston (Osnabrück University)

M14 | 19:00-19:20 | 0.5
Decomposition loci of tensors
Pierpaola Santarsiero (University of Bologna)

Tuesday 14	**Martes 14**	**Asteartea 14**
15:00-15:20	**15:00-15:20**	**15:00-15:20**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

SAGBI detection and SAGBI homotopy

Viktoriia Borovik
(Osnabrück University)

We introduce a SAGBI detection algorithm that, given a finite set of polynomials, finds all term orders for which they form a SAGBI basis. Using this tool, we implement a SAGBI homotopy feature in HomotopyContinuation.jl that provides a particular choice of a start system for homotopy continuation. For systems, where each equation is a linear combination of fixed polynomials, SAGBI homotopies significantly reduce the number of paths that need to be tracked, compared to polyhedral homotopies.

Joint work with E.Shehu and T.Duff, and ongoing joint work with B.Betti.

arXiv:2404.16796

Tuesday 14	**Martes 14**	**Asteartea 14**
15:30-15:50	**15:30-15:50**	**15:30-15:50**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Singularities of spectrahedra

Khazhgali Kozhasov
(Université Côte d’Azur)

Spectrahedra are convex sets that are defined via linear matrix inequalities (LMIs) and can be thought of as nonlinear generalizations of polyhedra. Typically, the topological boundary of a spectrahedron is not smooth. In my talk I will discuss a classification of ”combinatorial types” of spectrahedra defined by LMIs of size 5x5. Roughly speaking, a combinatorial type is the number of singular points on the boundary of the spectrahedron. The proof exploits tools of numerical algebraic geometry.

Joint work with Taylor Brysiewicz and Mario Kummer.

arXiv:2011.13860

Tuesday 14
16:00-16:20
[Room 0.5]

Martes 14
16:00-16:20
[Aula 0.5]

Asteartea 14
16:00-16:20
[Gela 0.5]

Generalized real monodromy

Margaret H Regan
(College of the Holy Cross)

The monodromy group is a geometric invariant that encodes the solution structure for a parameterized family of polynomial systems. An approach over the real numbers is the real monodromy structure, which contains tiered characteristics of the real solution set. This work generalizes the real monodromy structure to give information about partial permutations of real solutions induced by loops in an open subset of the parameter space. The talk will use motivating examples in optimization.

Joint work with Timothy Duff.

Tuesday 14
16:30-16:50
[Room 0.5]

Martes 14
16:30-16:50
[Aula 0.5]

Asteartea 14
16:30-16:50
[Gela 0.5]

Effective homology and numerical integration of periods

Eric Pichon-Pharabod
(MPI MiS Leipzig)

The period matrix of a smooth complex projective variety X encodes the isomorphism between its singular homology and its algebraic De Rham cohomology. Numerical approximations with sufficient precision of the entries of this matrix, called periods, allow to recover some algebraic invariants. We will present a method relying on the computation of an effective description of the homology for obtaining such numerical approximations of the periods of hypersurfaces and elliptic surfaces.

Tuesday 14	**Martes 14**	**Asteartea 14**
17:30-17:50	**17:30-17:50**	**17:30-17:50**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Mukai lifting of self-dual points in $\mathbb{P}^6$

Leonie Kayser
(MPI MiS Leipzig)

Motivated by Mukai's work on canonical curves, Petrakiev showed that a general self-dual set of 14 points in $\mathbb{P}^6$, invariant under the Gale transform, arises as the intersection of the Grassmannian Gr$(2, 6)$ in $\mathbb{P}^{14}$ with a linear space of dimension 6. In this paper we focus on the inverse problem of recovering such a linear space associated to a general self-dual set of points. We use numerical homotopy continuation to approach the problem and implement an algorithm in Julia to solve it.

Joint work with Barbara Betti.

arXiv:2406.02734

Tuesday 14	**Martes 14**	**Asteartea 14**
18:00-18:20	**18:00-18:20**	**18:00-18:20**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Preconditioning via Geodesically Convex Optimization

M. Levent Doğan
(Ruhr Universität at Bochum)

The computational complexity of iterative methods in numerical algebra generally depends on the "condition number". Preconditioning is a common method, whose goal is to reduce the condition number. Surprisingly, it is not known whether common preconditioners actually reduce the condition number. We will show that the problem of minimizing the condition number can be cast as a geodesically convex optimization problem. This framework leads to fast methods to compute optimal preconditioners.

Joint work with Alperen Ali Ergür.

Tuesday 14	**Martes 14**	**Asteartea 14**
18:30-18:50	**18:30-18:50**	**18:30-18:50**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Typical ranks of random order-three tensors

Sarah Eggleston
(Osnabrück University)

We study typical ranks of real $m \times n \times \ell$ tensors. For $(m-1)(n-1) < \ell < mn+1$ the typical ranks are contained in $\{\ell, \ell+1\}$, and l is always a typical rank; we provide a geometric proof. We express the probabilities of these ranks in terms of the probabilities of the numbers of intersection points of a random linear space with the Segre variety. For $m = n = 3$, the typical ranks of real $3 \times 3 \times 5$ tensors are 5 and 6; we link the rank probabilities to the probability of a random cubic surface having real lines.

Joint work with Paul Breiding and Andrea Rosana.

arXiv:2407.08371

Tuesday 14	**Martes 14**	**Asteartea 14**
19:00-19:20	**19:00-19:20**	**19:00-19:20**
[Room 0.5]	**[Aula 0.5]**	**[Gela 0.5]**

Decomposition loci of tensors

Pierpaola Santarsiero
(University of Bologna)

The decomposition locus of a tensor is the set of rank-one tensors appearing in a minimal tensor-rank decomposition of the tensor. In this talk, I will introduce this object, exploring its significance and trying to gain insights into the geometric and algebraic structures that govern tensor decompositions. I will highlight special examples of these loci, including the case of tangential tensors and pencil of tensors having maximal rank.

Joint work with Alessandra Bernardi and Alessandro Oneto.

arXiv:2407.18138